

CSC 347 - Concepts of Programming Languages

Algebraic Data Types

Instructor: Stefan Mitsch



Learning Objectives

- ❓ Complex data in functional programming?
 - Identify sums and products in algebraic data types
 - Use Scala `enum` classes to express algebraic data types



Algebraic Data Types

- Algebraic data types: sum of products

Product types

- Combine multiple data elements into one unit
- Tuples and classes

```
1 val product =  
2   (1, "hello", List(1, 2, 3))
```

Sum types

- Distinguish multiple alternatives
- Discriminated union, variants, subclasses

```
1 class A  
2 class B extends A  
3 class C extends A
```

- [Algebraic Data Types \(wikipedia\)](#) in PLs



Product Types

- Named for *Cartesian product* of sets
 - $X \times Y = \{(x, y) \mid x \in X \wedge y \in Y\}$

- Case class definition for product of `Int` and `String`

```
1 case class C(x:Int, y:String)
```

- Construct instances (`new` unnecessary)

```
1 val c:C = C(5, "hello")
```

- Extract elements with pattern matching

```
1 val n:Int = c match  
2   case C(a, b) => a
```



Scala Case Classes

- Compiler treatment for `case` classes
- Class parameters are visible and immutable (`val`)

```
1 case class C (x:Int, y:String)
2 val c:C = C (5, "hello")
3 val a:Int = c.x
4 c.x = 6 // error: reassignment to val
```

- Sensible `toString` implementation
- Companion object with `apply` method
 - used to construct instances
- Pattern matching support
 - see `unapply` method / extractors in textbook



Sum Types

- Named for sums (union) of sets

$$X \cup Y = \{z \mid z \in X \vee z \in Y\}$$

- *Coproduct* or *disjoint union* of sets

$$X \oplus Y = X \uplus Y = \{(0, x) \mid x \in X\} \cup \{(1, y) \mid y \in Y\}$$

- Elements are tagged to indicate their source (the class of the element)



Disjoint Union: Scala Enum

- Scala `enum` lists disjoint alternatives

```
1 enum Color:  
2   case Blue  
3   case White
```

- Disjoint union of value and operator

```
1 enum Expr:  
2   case Number(x: Int)  
3   case Plus(l: Expr, r: Expr)  
4   ...
```

- Definition can be recursive

- Create instances

```
1 val b = Color.Blue
```

```
1 import Expr.*  
2 val three = Number(3)  
3 val plus = Plus(three, Number(5))
```



Disjoint Union: Scala Enum

- Pattern match to decompose

```
1 def eval(e: Expr) : Int = e match
2   case Number(x)      => x
3   case Plus(l, r)     => eval(l) + eval(r)
4   ...
5 end eval
```

- Create instance and pass to `eval`

```
1 // (3+5)*2
2 val v = Times(Plus(Number(3), Number(5)), Number(2))
3 val i: Int = eval(v) // i==16
```



Exercise: Linked List

? Which "states" does a list have → which case classes and case objects?

- An `Empty` list and a `Cons` cell of at least one element

```
1 enum MyList:  
2   case Empty  
3   case Cons (head:Int, tail:MyList)
```



Exercise: Linked List

```
1 enum MyList:  
2   case Empty  
3   case Cons (head:Int, tail:MyList)
```

❓ Create an empty list?

- Simply use `Empty`

```
1 import MyList.*  
2 val xs = Empty
```

• ❓ Create an instance of a list?

- Nest `Cons` and terminate with `Empty`

```
1 import MyList.*  
2 val xs = Cons (1, Cons(2, Cons(3, Empty)))
```



Exercise: Linked List

```
1 enum MyList:
2   case Empty
3   case Cons (head:Int, tail:MyList)
4 object MyList:
5   def create(elems: Int*) : MyList =
6     if elems.isEmpty then Empty
7     else Cons(elems.head, create(elems.tail*))
```

- ? Create an instance of a list?
- Use method `create`

```
1 import MyList.*
2 // val xs = Cons (1, Cons(2, Cons(3, Empty)))
3 val xs = MyList.create(1, 2, 3)
```



Exercise: Linked List

? Generalize to any element type?

```
1 enum MyList[+X]:  
2   case Empty  
3   case Cons (head:X, tail:MyList[X])
```

? Compute the length of such a list?

- Recursive with pattern matching

```
1 def length [X] (xs:MyList[X]): Int = xs match  
2   case MyList.Empty      => 0  
3   case MyList.Cons(a,as) => 1 + length(as)
```



Summary

Algebraic data types: sum of products

Product types

- Combine multiple data elements
- Tuples and classes

```
1 val product = (1, "hello", List(1, 2, 3))
```

Sum types

- Distinguish multiple alternatives
- Subclasses

```
1 class A; class B extends A; class C extends A
```

- In Scala: `enum` and `case` classes

```
1 enum Sum:  
2   case Product1(i: Int, s: String)  
3   case Product2(i: Int, j: Int, k: Int)  
4   ...
```



Exercise: Peano Natural Numbers

- Peano natural numbers: either 0 or a transitive successor of it
- Algebraic data type `PeanoNat`

```
1 enum PeanoNat:  
2   case Zero  
3   case Succ(n: PeanoNat)
```

- Evaluate `PeanoNat` to `Int`

```
1 import PeanoNat.*  
2 def peano2int (p: PeanoNat, result: Int = 0): Int = p match  
3   case Zero      => result  
4   case Succ(n) => peano2int (n, result+1) // tail-recursive  
5  
6 val q = Succ(Succ(Succ(Zero))) // val q: Peano = ...  
7 peano2int(q) // : Int = 3
```



Exercise: Binary Operations Tree

- Data stored at leaves
- Operations stored at internal nodes
- Internal nodes have left and right subtrees
- Algebraic data type

```
1 enum Tree[X]:  
2   case Leaf (data:X)  
3   case Node (l:Tree[X], f:(X,X)=>X, r:Tree[X])
```

- ? Fold tree into result by applying all the intermediate operations
- Recursive with pattern matching

```
1 def fold [X] (t: Tree[X]) : X = t match  
2   case Leaf(x) => x  
3   case Node(l, f, r) => f(fold(l), fold(r))
```