

CSC 347 - Concepts of Programming Languages

Subtyping

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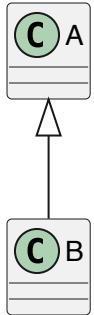


Learning Objectives

- ❓ What should the type relationship between parameterized types be?
 - Identify read and write restrictions on parametric types



Subtyping



- Static and dynamic type

```
1 A x = new A ();
2 B y = new B ();
3 x = y; // B ok when A expected
```

- Method parameters

```
1 void aConsumer (A x) { ... }
2 aConsumer (new B()); // B ok when A expected
```

- Method results

```
1 B bProducer () { ... }
2 A x = bProducer (); // B ok when A expected
```

- Safe to use an instance of B when an A is expected
- B is a *subtype* of A : written $B <: A$
 - If $y:B$ and $B <: A$ then $y:A$ (upcast)
- Subtyping is not just subclassing: parametric polymorphism



Subtyping Order: Top

- Subtyping relation `<:` is a *partial order* on types
 - *reflexive*: `X<:X`
 - *transitive*: if `Duck<:Bird` and `Bird<:Animal` then `Duck<:Animal`
- Some PLs have a `Top` type: `X<:Top` for all `X` (greater than all other types)
- In Java: `java.lang.Object` above reference types
- In Scala: `scala.Any` above all types, `scala.AnyRef` above reference types

```
1 val xs:List[AnyRef] = List ("hello", List(1))
2 val ys:List[Any]    = List ("hello", 1)
```



Subtyping Order: Bottom

- Most PLs do not have a `Bottom <: X` for all `X` (less than all other types)
- In [Scala Type Hierarchy](#): `Nothing` is `Bottom` type (empty, no instances)
- ? What is `Bottom` useful for?
- Important for typing neutral elements of parameterized types, e.g. `Nil`

```
1 val nil:List[String] = Nil
2 // Best type possible
3 val xs1:List[String] = "hello" :: nil
4 val xs2:List[Any]    = 1 :: nil
```

```
1 val nil:List[Nothing] = Nil
2 // Best type possible
3 val xs1:List[String] = "hello" :: nil
4 val xs2:List[Int]    = 1 :: nil
```

- `Nil : List[Nothing]` and `List[Nothing] <: List[X]`, so
`(X :: List[Nothing]) : List[X]`



Type-Invariant Containers

```
1 enum Animal:  
2   case Bird(name: String)  
3   case Cat(name: String)  
4   def name: String  
5 end Animal  
6  
7 case class Box[X](var content: X)
```

- Create concrete boxes

```
1 val birdBox = Box(Bird("Feathers"))  
2 val catBox  = Box(Cat("Fluffy"))
```

- Create an abstract box

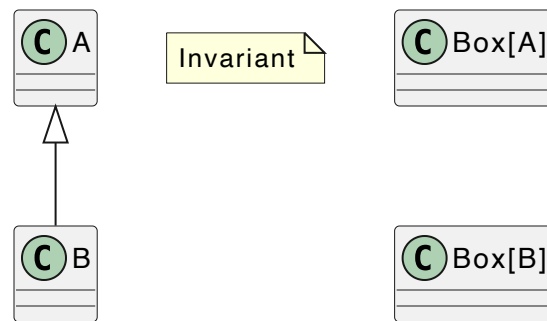
```
1 var animalBox = Box[Animal](Bird("Feathers"))  
2 animalBox.content = Cat("Fluffy")
```

- Alias boxes

```
1 animalBox = birdBox // does not compile
```

? Why not `animalBox = birdBox` ?

- Does not compile because `Box[Bird]` is not a subtype of `Box[Animal]`
- Box is **invariant** in its type parameter X





Type-Invariant Containers

? What would go wrong if we allow assignment?

```
1 val birdBox = Box(Bird("Feathers"))  
2 var animalBox: Box[Animal] = birdBox // does not compile, but let's assume it passes
```

```
1 val animal: Animal = animalBox.content // ok, we can read bird as an animal
```

```
1 animalBox.content = Cat("Fluffy") // we can put a cat into an animal box
```

```
1 val bird: Bird = birdBox.content // but now the bird box contains a cat!
```

! Writing to aliased container would create type violations at runtime!



Type-Covariant Containers

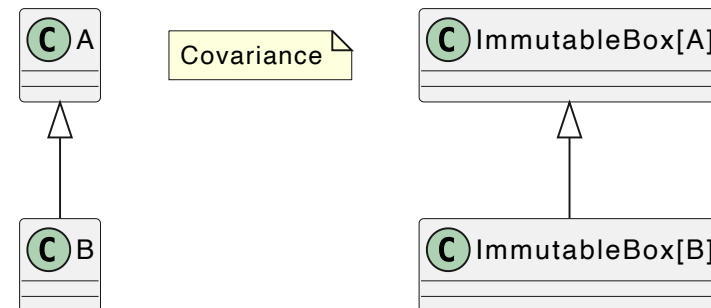
? What if we disallow modifying boxes?

```
1 case class ImmutableBox[+X](val content: X)
```

- Create and alias boxes

```
1 val birdBox = ImmutableBox(Bird("Feathers"))  
2 val animalBox: ImmutableBox[Animal] = birdBox // compiles  
3 val animal = animalBox.content                // reading ok  
4 animalBox.content = Cat("Fluffy") // immutable, cannot write
```

- `ImmutableBox` is **covariant** in `X`
- Covariance type annotation `+X` can only be applied to read-only structures





Covariant Boxes

? What is the practical relevance?

```
1 def printAnimalName(b: ImmutableBox[Animal]) =  
2   println(b.content.name)
```

- Print some concrete boxes

```
1 val birdBox = ImmutableBox(Bird("Feathers"))  
2 val catBox  = ImmutableBox(Cat("Fluffy"))  
3  
4 printAnimalName(birdBox)  
5 printAnimalName(catBox)
```



Scala Lists are Covariant

? What is the practical relevance?

```
1 def printAnimalNames(animals: List[Animal]) =  
2   for a <- animals do println(a.name)
```

- Print some concrete lists

```
1 val birds: List[Bird] = List(Bird("Feathers"), Bird("Chirpy"))  
2 val cats: List[Cat]   = List(Cat("Fluffy"), Cat("Whiskers"))  
3  
4 printAnimalNames(birds)  
5 printAnimalNames(cats)
```

- 💡 `List` must be covariant to allow `printAnimalNames` with concrete arguments of type `List[Bird]` and `List[Cat]` !



Scala Arrays are Invariant

```
1 def printAnimalNames(animals: Array[Animal]) =  
2   for a <- animals do println(a.name)  
3  
4 val birds: Array[Bird] = Array (Bird("Feathers"), Bird("Chirpy"))  
5 printAnimalNames(birds)
```

- Compile error

```
1 printAnimalNames(birds)  
2           ^^^^^  
3           Found: (birds : Array[Bird])  
4           Required: Array[Animal]
```

- Scala and Java arrays are readable and writable
 - Scala arrays are invariant: compile error when writing to alias
 - Java arrays are covariant: runtime exception when writing to alias



Covariant Wildcard

❓ How can we still print arrays generically?

- Use a covariant wildcard to make array read-only

```
1 def printAnimalNames(animals: Array[? <: Animal]) =  
2   for a <- animals do println(a.name)  
3  
4 val birds: Array[Bird] = Array (Bird("Feathers"), Bird("Chirpy"))  
5 printAnimalNames(birds) // compiles
```

- Cannot write to `Array[? <: Animal]`

```
1 animals(0) = Cat("Fluffy")  
2           ^^^^^^^^^^^^^^^  
3           Found:    Cat  
4           Required: animals.T
```



What about Writing?

```
1 def putCat(box: Box[Cat]) =  
2   box.content = Cat("Fluffy")
```

- Unable to pass in a box of animal

```
1 val animalBox: Box[Animal] = Box(Cat("Whiskers"))  
2 putCat(animalBox)  
3     ^^^^^^^  
4     Found:    (animalBox : Box[Animal])  
5     Required: Box[Cat]
```

- Because the animal box could be a bird box

```
1 val animalBox: Box[Animal] = Box(Bird("Feathers"))  
2 putCat(animalBox)
```



Contravariance

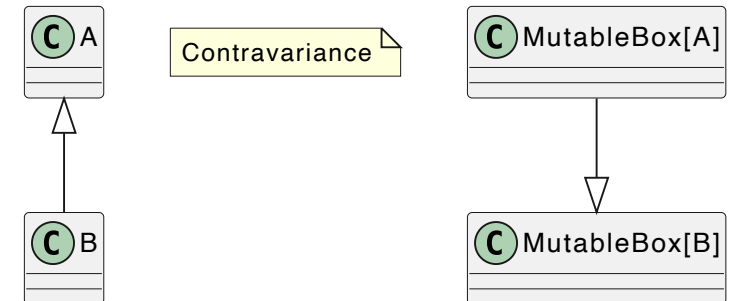
? What if we disallow reading?

```
1 class MutableBox[-X](content: X):  
2   private var x: X = content  
3   def put(elem: X) : Unit = x = elem
```

- Create and alias boxes

```
1 def putCat(box: MutableBox[Cat]) = box.put(Cat("Fluffy"))  
2 val animalBox: MutableBox[Animal] = new MutableBox(Cat("Whiskers"))  
3 putCat(animalBox)
```

- `MutableBox` is **contravariant** in `X`
- Contravariance type annotation `-X` can only be applied to write-only structures





Contravariance allows Degraded Read

```
1 class MutableBox[-X](content: X):  
2   private var x: X = content  
3   def put(elem: X) : Unit = x = elem  
4   def get : Any = x  
5  
6 def putCat(box: MutableBox[Cat]) = box.put(Cat("Fluffy"))
```

- Create box and modify it

```
1 val animalBox: MutableBox[Animal] = new MutableBox(Bird("Feathers"))  
2 var animal: Any = animalBox.get  
3  
4 putCat(animalBox)  
5 animal = animalBox.get
```



Contravariant Wildcard

```
1 def addFluffy(cats: List[Cat]) =  
2   Cat("Fluffy") :: cats
```

- Add to a list of cats

```
1 val cats: List[Cat] = List(Cat("Whiskers"))  
2 val moreCats: List[Cat] = addFluffy(cats) // ok
```

- Add to a list of animals

```
1 val animals: List[Animal] = List(Bird("Feathers"), Cat("Whiskers"))  
2 addFluffy(animals)  
3           ^^^^^^^  
4           Found:    (animals : List[Animal])  
5           Required: List[Cat]
```



Contravariant Wildcard

```
1 def addFluffy(cats: List[? >: Cat]) =  
2   Cat("Fluffy") :: cats
```

- Contravariant wildcard `? >: X`
- Add to a list of animals

```
1 val animals: List[Animal] = List(Bird("Feathers"), Cat("Whiskers"))  
2 val more: List[Any] = addFluffy(animals)
```

- ? Can we do better than `List[Any]` ?



Contravariant Type Parameter

- Use a contravariant type parameter rather than a wildcard

```
1 def addFluffy[X >: Cat](cats: List[X]) =  
2   Cat("Fluffy") :: cats
```

- Add to a list of animals

```
1 val animals: List[Animal] = List(Bird("Feathers"), Cat("Whiskers"))  
2 val moreAnimals: List[Animal] = addFluffy(animals)
```



Unusual Use Case: Programming with Types

Peano arithmetic in types

```
1 trait Nat
2 class _0 extends Nat
3 class Succ[A <: Nat] extends Nat
4
5 type _1 = Succ[_0]
6 type _2 = Succ[_1]
7 // ...
```

Comparison types

```
1 trait <[A <: Nat, B <: Nat]
```

```
1 object < {
2   // base case: 0 is less than any successor
3   given [B <: Nat] => <[_0, Succ[B]]
4   // inductive case: if A < B then Succ[A] < Succ[B]
5   given [A <: Nat, B <: Nat] => <[A, B] => <[Succ[A], Succ[B]]
6   // helper method to summon an instance (a proof)
7   def apply[A <: Nat, B <: Nat](using lt: <[A, B]) = lt
8 }
```

- Compiler infers type of `p` given types in `object <`

```
1 val p = <[_0, _1] // 0 < 1
2 // <[_1, _0] // 1 < 0 does not compile
```



Summary

- *Subtype polymorphism*: $B <: A$ (B is a subtype of A)
- *Parametric polymorphism*: $T[X]$ parameterize class T with type parameter X
- Covariant, contravariant, and invariant parametric types for $B <: A$

	Covariant	Contravariant	Invariant
	Readable	Writable	Read-write
Relationship	$T[B] <: T[A]$	$T[A] <: T[B]$	$T[A] <> T[B]$
Box	<code>ImmutableBox[X]</code>	<code>MutableBox[X]</code>	<code>Box[X]</code>
Collection	<code>List[X]</code>		<code>Array[X]</code>
Annotations	$T[+X]$	$T[-X]$	$T[X]$
Functions	$\text{Unit} \Rightarrow X$	$X \Rightarrow \text{Unit}$	$X \Rightarrow X$



Java Arrays are Covariant

```
1 public static void main (String[] args) {  
2     B[] bs = new B[] { new B (), new B () };  
3     A[] as = bs;          // OK, because covariant  
4     as[0] = new A (); // ArrayStoreException  
5     bs[0].g();  
6 }
```

```
1 $ javac Driver.java  
2 $ java Driver  
3 Exception in thread "main" java.lang.ArrayStoreException: A  
4   at Driver.main(Driver.java:5)
```

- Every assignment to object array dynamically checked!



Why?

- For example, to sort

```
1 static void sort(Object[] xs) { ... }  
2 String[] ss = ...;  
3 sort(ss); // requires covariance
```

- With Java Generics

```
1 static <X extends Comparable<? super X>> void sort(X[] xs) { ... }
```

- In Scala

```
1 def sort[X <: Comparable[? >: X]] (xs: Array[X]) = ...
```

- Use

```
1 class A implements Comparable<A>{}  
2 class B extends A {}  
3 B[] bs = ...;  
4 sort(bs); // B's are comparable as A's
```



Variance Annotations Example

```
1 trait Source[+X] { def get () : X } // Covariant: read-only
2 trait Sink [-X] { def put (x:X) : Unit } // Contravariant: write-only
```

```
1 class Ref [ X] (var contents:X) // Invariant: read-write
2 extends Source[X] with Sink[X]:
3   def get () = contents
4   def put (x:X) = contents = x
```

- Create aliases

```
1 val ref: Ref[Bird] = Ref(Bird("Feathers"))
2 val src: Source[Animal] = ref
3 val snk: Sink[Duck] = ref
```

- Write to `snk`, read from `ref` and `src`

```
1 val d = Duck("Quack")
2 snk.put(d)
3 val r = ref.get()
4 val s = src.get()
```

- What are the static and dynamic types of `d`, `r`, and `s`?



Variance Annotations Example

```
1 trait Function[-X,+Y] { def apply(x:X) : Y      } // Contravariant in X, covariant in Y
```

- Covariant use

```
1 // List[Int] <: Seq[Int], so Function[Int,List[Int]] <: Function[Int,Seq[Int]]
2 val increment: Function[Seq[Int],Seq[Int]] = new Function[Seq[Int],List[Int]]:
3   def apply(x: Seq[Int]) : List[Int] = x.map(_ + 1).toList
4 // Int <: Any, so Function[Seq[Int],Int] <: Function[Seq[Int],Any]
5 val length: Function[Seq[Int],Any] = new Function[Seq[Int],Int]:
6   def apply(x: Seq[Int]) : Int = x.length
```

- Contravariant use

```
1 def chain(f: Function[List[Int],Seq[Int]], g: Function[Seq[Int],Any]) =
2   (x:List[Int]) => g(f(x))
3 chain(increment, length)
```



Variance Annotations

```
1 trait Producer[+Y]      { def apply ()      : Y      } // Covariant
2 trait Consumer[-X]      { def apply (x:X)    : Unit   } // Contravariant
3 trait Function[-X,+Y]   { def apply (x:X)    : Y      } // Both
4 trait Operator[X]       { def apply (x:X)    : X      } // Invariant
```

```
1 trait Producer[-Y]      { def apply ()      : Y      }
2                          ^
3 error: contravariant type Y occurs in covariant position
```

```
1 trait Consumer[+X]      { def apply (x:X)    : Unit   }
2                          ^
3 error: covariant type X occurs in contravariant position
```